Fermionic functional renormalization group for first-order phase transitions

A mean-field model

Roland Gersch, Julius Reiss, Carsten Honerkamp

Dept. Metzner, MPI for Solid State Research, Stuttgart

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Outline

The fRG can scan a system's order parameter space for minima of the thermodynamic potential.

1. Some fRG
2. Bias as a challenge
3. Bias as a chance

Motivation

- Spontaneous symmetry breaking

$$\sum_X \Psi_X \mathbb{Q}(X) \Psi_X \stackrel{\text{i.a.}}{\to} \sum_{XY} \Psi_X (\mathbb{Q}(X) - \Delta(X,Y)) \Psi_Y$$

- Microscopic models $\stackrel{\text{fRG}}{\to}$ physical observables
- Fermionic degrees of freedom kept
- Unbiased approach for multiple instabilities

  Zanchi and Schulz 1998, Halboth and Metzner 2000, Salmhofer, Honerkamp, Furukawa, Rice 2001, Metzner, Reiss, Rohe 2005, Dupuis 2005.
- Rigorous error estimates available

  Salmhofer, Honerkamp 2001

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- Generating functional of connected Green's functions

$$\exp(-W(J)) = \int \frac{\mathcal{D}\Psi}{\det \mathbb{Q}/\chi(\Lambda)} e^{(\Psi,\mathbb{Q}/\chi(\Lambda)\Psi)-V(\Psi)+(J,\Psi)}$$

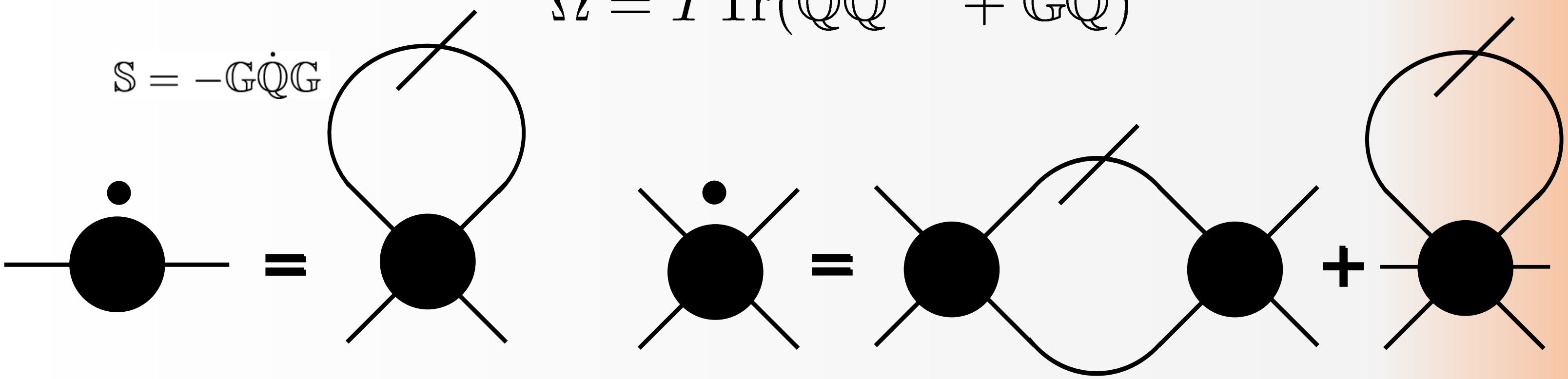
J: Grassmannian generating field, V: two-particle interaction

- $\chi(\Lambda_{\mathrm{i}})$ makes W solvable, $\chi(\Lambda_{\mathrm{f}}) \equiv 1$.

One-particle irreducible (1PI) flow Salmhofer, Honerkamp 2001

$$\dot{\Omega} = T\mathrm{Tr}(\dot{\mathbb{Q}}\mathbb{Q}^{-1} + \mathbb{G}\dot{\mathbb{Q}})$$

$\mathbb{S} = -\mathbb{G}\dot{\mathbb{Q}}\mathbb{G}$



Formalities

- Generating functional of connected Green's functions

$$\exp(-W(J)) = \int \frac{\mathcal{D}\Psi}{\det \mathbb{Q}/\chi(\Lambda)} e^{(\Psi,\mathbb{Q}/\chi(\Lambda)\Psi)-V(\Psi)+(J,\Psi)}$$

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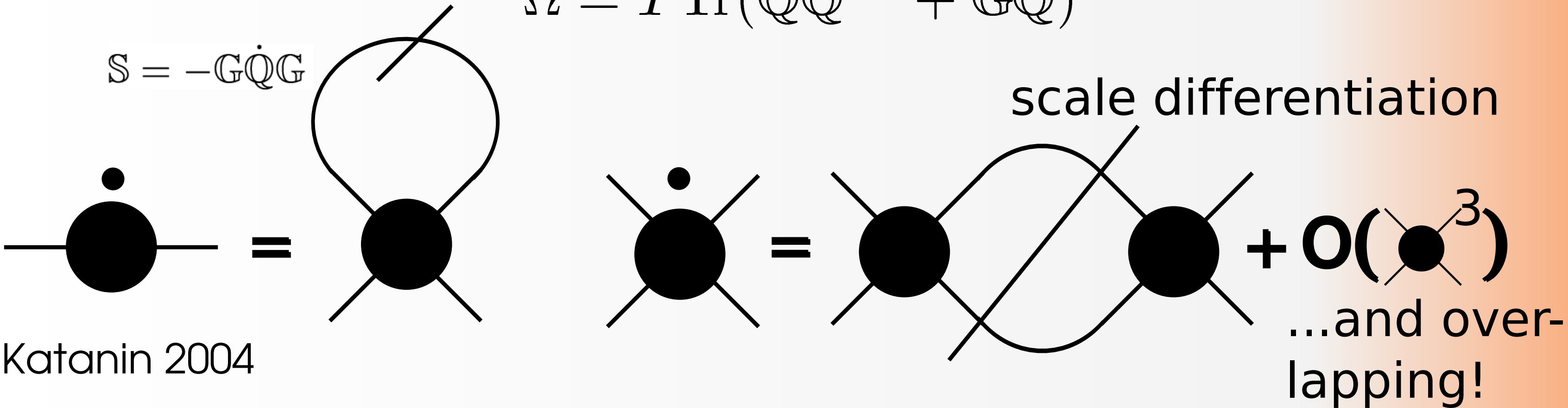


Katanin 2004

The external field's influence

Charge-density-wave mean-field model Hamiltonian

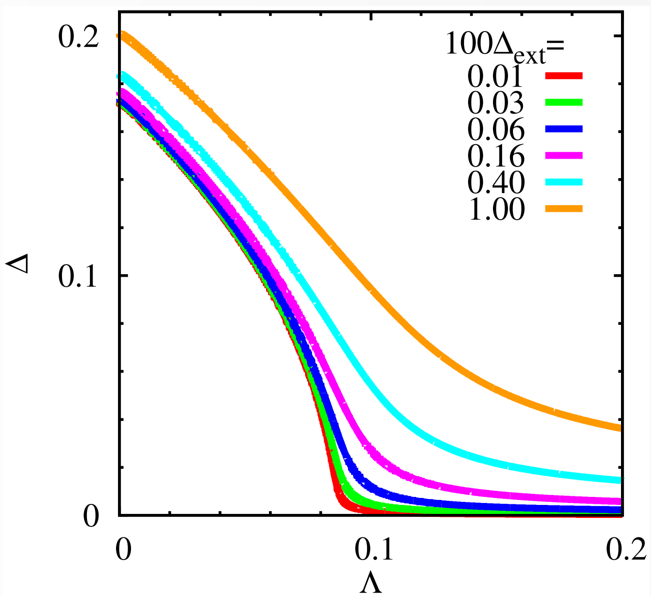
$$H = \sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \mu) c^\dagger_{\mathbf{k}} c_{\mathbf{k}} - \frac{V_0}{N} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}} c_{\mathbf{k}+\mathbf{Q}} c^\dagger_{\mathbf{k}'} c_{\mathbf{k}'+\mathbf{Q}}$$
$$+ \sum_{\mathbf{k}} (\Delta_c - \Delta_i) c^\dagger_{\mathbf{k}+\mathbf{Q}} c_{\mathbf{k}}, \quad \mathbf{Q} = (\pi, \pi, \dots)$$

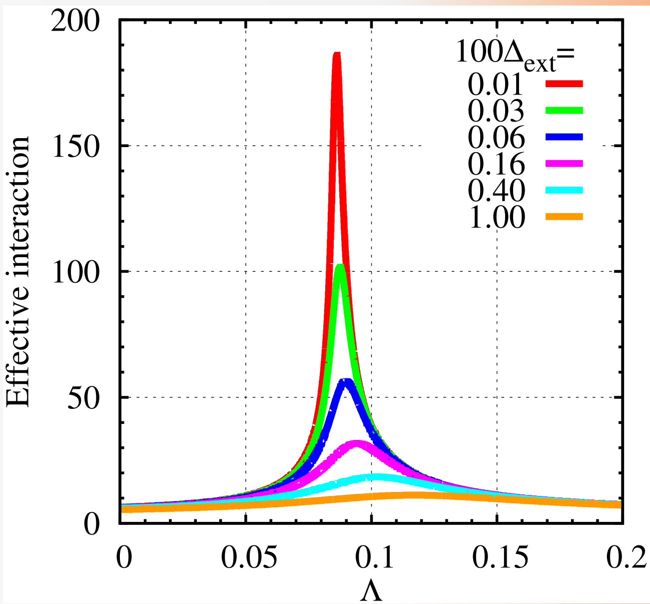
μ: chemical potential, ε: tight-binding dispersion, V_0: nearest-neighbor repulsion, $\Delta_c - \Delta_i$: external field.

$T = \mu = 0:$

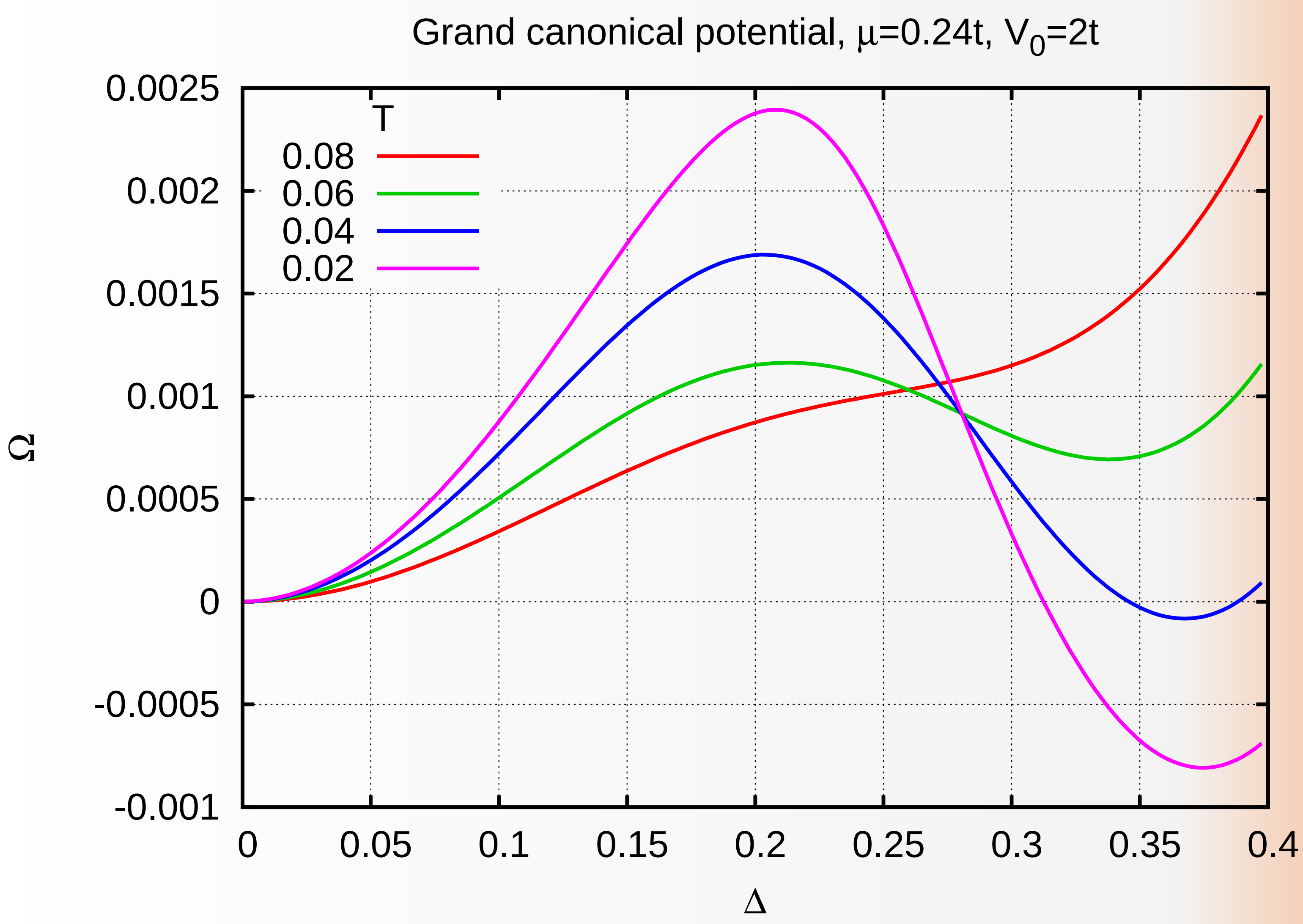
RG, Honerkamp, Rohe, Metzner 2005

Units: hopping integral





First-order phase transitions



Counterterms: first attempt

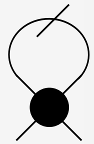
- Back to the CDW Hamiltonian.

$$H = \sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \mu) c^\dagger_{\mathbf{k}} c_{\mathbf{k}} - \frac{V_0}{N} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}} c_{\mathbf{k}+\mathbf{Q}} c^\dagger_{\mathbf{k}'} c_{\mathbf{k}'+\mathbf{Q}}$$

$$+ \sum_{\mathbf{k}} (\Delta_{\mathrm{c}} - \Delta_{\mathrm{i}}) c^\dagger_{\mathbf{k}+\mathbf{Q}} c_{\mathbf{k}}$$

- To bare propagator
- To initial self-energy

$$\mathbb{G}^{-1} = \frac{1}{\chi} \begin{pmatrix} i\omega - \varepsilon + \mu & \Delta_{\mathrm{c}} - \chi\Delta \\ \Delta_{\mathrm{c}} - \chi\Delta & i\omega + \varepsilon + \mu \end{pmatrix}$$

$$\Rightarrow \mathbb{G}_{12} \propto \chi \cdot \underbrace{(\chi\Delta - \Delta_{\mathrm{c}})}_{=:-\Delta_{\mathrm{eff}}} \overset{\chi(\Lambda)\in\{0,1\}}{\Rightarrow} \dot{\Delta} =$$  $$\propto \chi \cdot (\chi\Delta - \Delta_{\mathrm{c}}) = 0$$

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Interaction flow

$\chi(\Lambda) \equiv \sqrt{\Lambda}$, $\Lambda_\mathrm{i} = 0$, $\Lambda_\mathrm{f} = 1$ equivalent to linearly turning on the interaction from 0 to V_0.

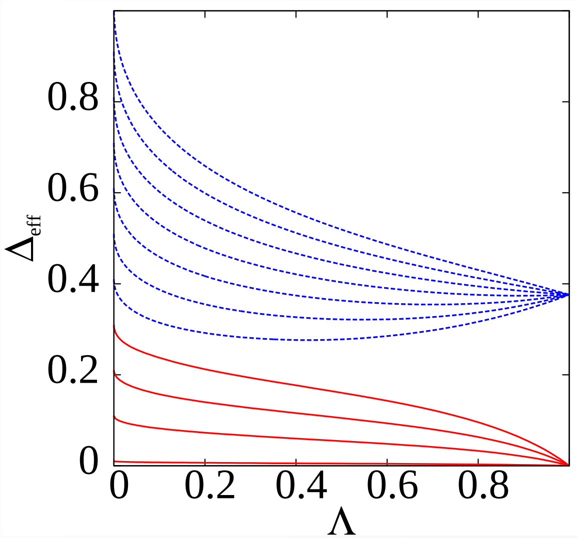
Honerkamp, Rohe, Andergassen, Enss 2004.

Advantages:

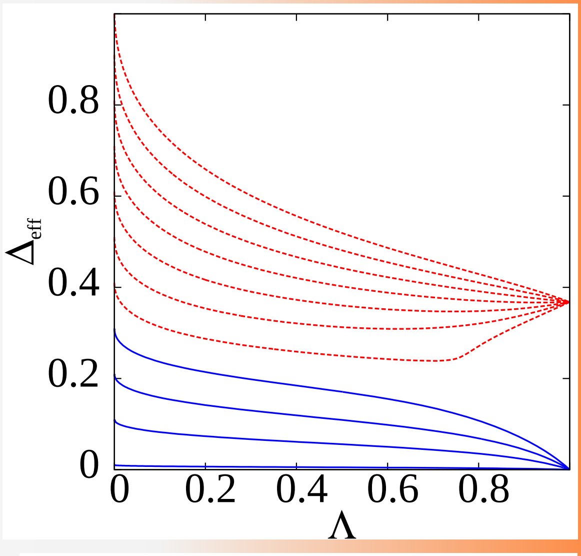
- $\chi\Delta_\mathrm{eff}|_{\Lambda=\delta\Lambda} \approx \sqrt{\delta\Lambda}(-\Delta_\mathrm{c}) \neq 0$
- Thus, $\dot{\Delta} \neq 0$.
- Chosing $\Delta_\mathrm{i} = \Delta_\mathrm{c}$: scanning the order parameter space for thermodynamic potential minima.
- Results independent of Δ_i for mean-field models
- Large effective interactions restricted to the final region of the flow

First-order CDW phase transition

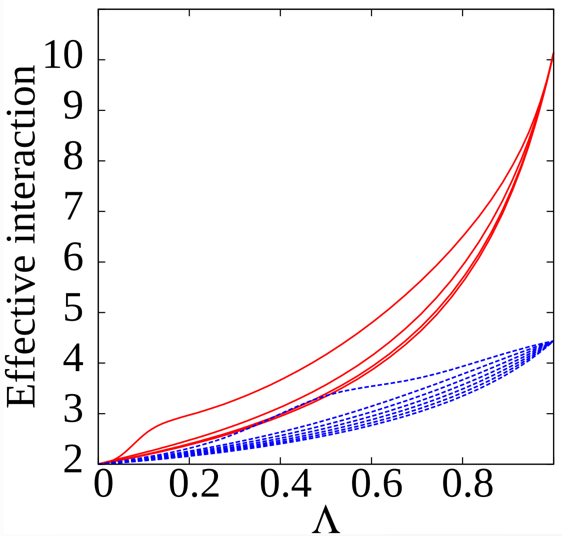
Order parameter
$T < T_t$
units: hopping integral



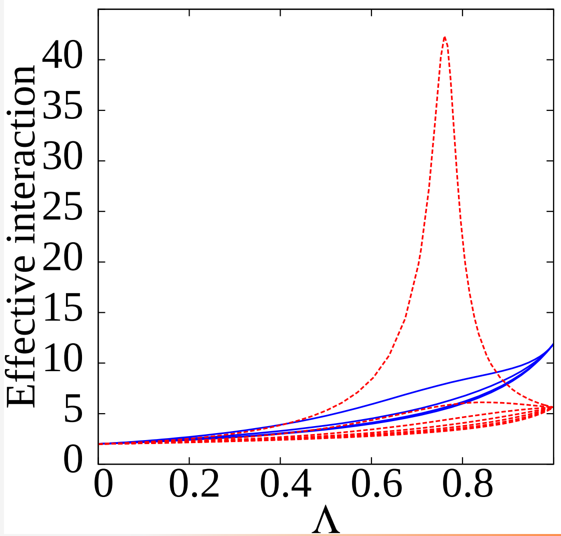
$T > T_t$



Effective interaction
$T < T_t$
units: hopping integral



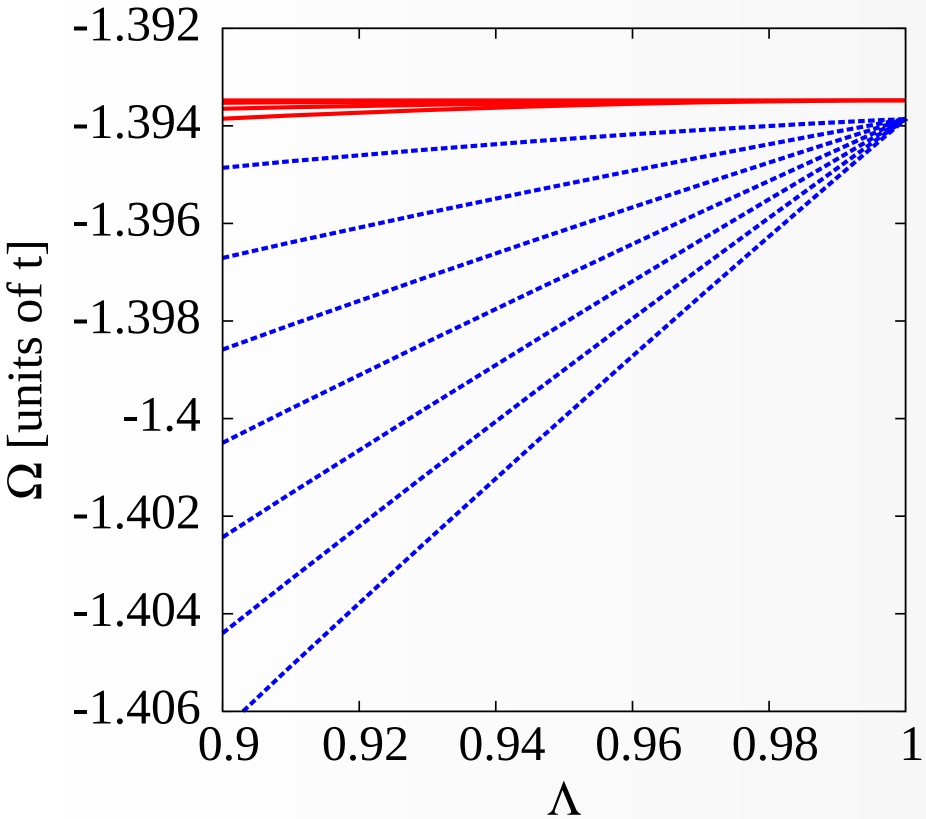
$T > T_t$



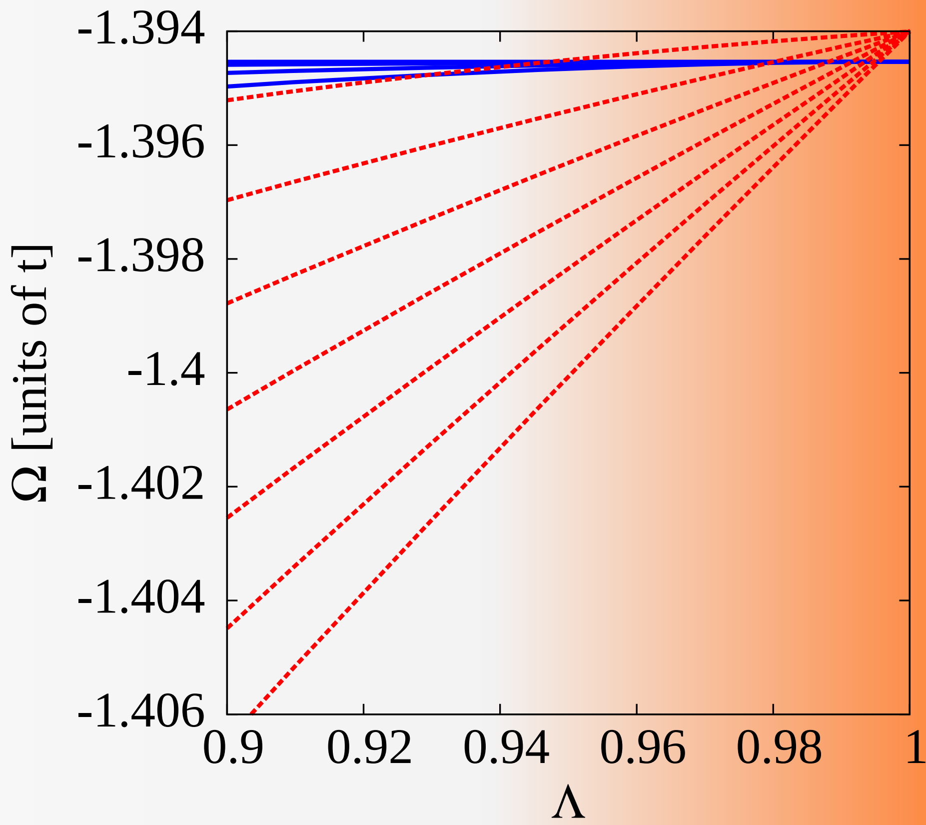
RG, Reiss, Honerkamp 2006

Flow of Ω

Ω for $T < T_t$:



Ω for $T > T_t$:



RG, Reiss, Honerkamp 2006

Conclusions

- The fRG is set up as a powerful tool for the study of symmetry breaking.
- A method to study stable and metastable states has been developed. Bias has turned from challenge to chance.
- Studies of models with extended momentum structure remain to be done (discretization: patching, expansion, ...?)

Thank you very much!

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